Benha University Faculty of Engineering – Shoubra Department of Elec. Engineering & Control Duration : 2 hours	Final Exam Course: Mathematics 1 Code: EMP 101 Date: December 23 , 2017

The exam consists of one page   No. of questions: 4   Answer **All** questions   Total Mark: 40

Question 1

(a)Find **y`** from the following : 6

(i) $y = 2x^3 - 2^x + x$ (ii) $y = e^x . \tan x$ (iii) $y = \log x - \ln x$

(iv) $y = \frac{1}{x^3} + \ln(x^2 + 1)$ (v) $y = \cos x . \ln x$ (vi) $y = (x + \sin x)^5$

(b) Determine the maximum and minimum points of : $f(x) = x^4 - 8x^2 + 1$ 3

(c) Write the Maclurin's expansion of : $f(x) = (x + 2)\sin x$ 3

Question 2

Find the integrals: 6

(a) $\int (x^2 + 2^x - 2)\, dx$ (b) $\int (1 - x^2)^2\, dx$ (c) $\int (\frac{1}{x^3} + e^x)\, dx$

(d) $\int (\sqrt{x} + \cos x)\, dx$ (e) $\int (x + \frac{1}{x})^2\, dx$ (f) $\int (x - 3\sin 2x)\, dx$

Question 3

(a) If $A = \begin{bmatrix} 0 & -3 & 2 \\ 1 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 2 \end{bmatrix}$ 8

Find, if possible, A + B, A + C, A.B , A^t.B, |A| , |B.C|.

(b) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. 4

Question 4

(a)Solve the linear systems : 2

(i) 2x + y + z = 6, – x + 2y – 3z = – 3, x + 2y – 2z = 2. 3

(ii) x + y + z = 6, – 2x + y – 3z = 3, – x + 2y – 2z = 8.

(b)Using the binomial theorem, expand : (i) $\frac{1}{\sqrt{1-2x}}$ (ii) $\frac{x+3}{x^2-3x+2}$ 2

(c)If $z_1 = 2 + 2i$, $z_2 = 1 - 3i$. Find $z_1 + z_2$, $z_1 . z_2$, $(z_1)^5$, $\sqrt[3]{z_1}$. 3

Good Luck   *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)(i) $y` = 6x^2 - 2^x.\ln 2 + 1$

(ii) $y` = e^x.\tan x + e^x.\sec^2 x$

(iii) $y` = \frac{1}{\ln 10}.\frac{1}{x} - \frac{1}{x}$

(iv) $y` = -3x^{-4} + \frac{2x}{x^2+1}$

(v) $y` = \cos x.\frac{1}{x} - \sin x.\ln x$

(vi) $y` = 5(x + \sin x)^4.(1 + \cos x)$

--*6 Marks*

(b) Since $f`(x) = 4x^3 - 16x = 0$. Then x = 0, x = 2, x = – 2.

From $f``(x) = 12x^2 - 16$.

We see that $f``(2) = 48 - 16 = 32$, then x = 2 is minimum.

We see that $f``(-2) = 48 - 16 = 32$, then x = – 2 is minimum.

We see that $f``(0) = 0 - 16 = -16$, then x = 0 is maximum.

--*3 Marks*

(c) Since $f(0) = (0 + 2)\sin 0 = 0$

$f`(x) = (x + 2)(\cos x) + 1\sin x$, then $f`(0) = 2$

$f``(x) = (x + 2)(-\sin x) + 2\cos x$, then $f``(0) = 2$

$f```(x) = (x + 2)(-\cos x) - 3\sin x$, then $f``(0) = -2$

Then $f(x) = 0 + 2x + \frac{2}{2!}x^2 - \frac{2}{3!}x^3 + \cdots$

--*3 Marks*

Answer of Question 2

(a) $I = \frac{1}{3}x^3 + \frac{1}{\ln 2}2^x - 2x + c$

(b) $I = \int(1 - 2x^2 + x^4)\,dx = x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + c$

(c) $I = \frac{x^{-2}}{-2} + e^x + c$

(d) $I = \frac{2}{3}x^{\frac{3}{2}} + \sin x + c$

(e) $I = \int(x^2 + 2 + \frac{1}{x^2})\,dx = \frac{1}{3}x^3 + 2x - x^{-1} + c$

(f) $I = \frac{1}{2}x^2 + \frac{3}{2}\cos 2x + c$

--*6 Marks*

Answer of Question 3

(a) A + C, A.B, |A| are impossible.

$A + B = \begin{bmatrix} 1 & -3 & 6 \\ 3 & 0 & 2 \end{bmatrix}$, $A`B = \begin{bmatrix} 2 & 2 & 1 \\ -7 & -4 & -14 \\ 4 & 2 & 9 \end{bmatrix}$, $BC = \begin{bmatrix} 6 & 9 \\ 5 & 10 \end{bmatrix}$ and $|BC| = 15$

--*8 Marks*

(b) $\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)(2-\lambda) - 1 = \lambda^2 - 4\lambda + 3 = 0$. Then $\lambda_1 = 1, \quad \lambda_2 = 3$

From the equation, $\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 1$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then x + y = 0,   x + y = 0

Then x = – y = any number except 0

Put y = 1, we get x = – 1 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

For : $\lambda_2 = 3$, $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then –x + y = 0,   x – y = 0

Then x = y = any number except 0

Put y = 2, we get x = 2 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

--*4 Marks*

Answer of Question 4

(a)(i)By Calculator, the solution is : (2, 1, 1).

--*2 Marks*

(a)(ii) $G = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ -2 & 1 & -3 & 3 \\ -1 & 2 & -2 & 8 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 3 & -1 & 15 \\ 0 & 3 & -1 & 14 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 3 & -1 & 15 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Rank A = 2 and Rank G = 3. No solution.

--*3 Marks*

(b)(i) $\frac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}} = 1 + x + \frac{3}{2}x^2 + \cdots, \quad |2x| < 1$

(ii) $\frac{x+3}{x^2-3x+2} = \frac{-4}{x-1} + \frac{5}{x-2} = 4(1-x)^{-1} - \frac{5}{2}(1-\frac{1}{2}x)^{-1}$

$$= 4[1 + x + x^2 + \cdots] - \frac{5}{2}[1 + \frac{1}{2}x + \frac{1}{4}x^2 + \cdots]$$

--*2 Marks*

(c) $z_1 + z_2 = 3 - \mathrm{i}, \quad z_1 . z_2 = 8 - 4i,$

Since $\mathrm{z}_1 = 2 + 2\mathrm{i} = \sqrt{8}\, \mathrm{e}^{\frac{\mathrm{i}\pi}{4}}$. Then $(\mathrm{z}_1)^5 = 8^{\frac{5}{2}}\, \mathrm{e}^{\mathrm{i}\frac{5\pi}{4}}$ and $(\mathrm{z}_1)^{\frac{1}{3}} = 8^{\frac{1}{6}}\, \mathrm{e}^{\mathrm{i}\frac{\pi}{12}}$
--*3 Marks*

Dr. Mohamed Eid